POLITECNICO di MILANO
DIPARTIMENTO di ENERGETICA

IMPLEMENTATION AND VALIDATION OF NLEVM IN CFD COMMERCIAL CODE

Emanuela Colombo, Davide Fortini, Fabio Inzoli, Riccardo Mereu
Politecnico di Milano, Dipartimento di Energetica, Piazza Leonardo da Vinci, 32 - 20133 MILANO

PURPOSE

The research field of the study is related with turbulence modeling. The general objective is the implementation of a two equations Non Linear Eddy Viscosity Models (NLEVM) for the Reynolds stress tensor in a CFD commercial code.
The model is a second order k-ε model based over Shih, Zhu and Lumley (1993) and Craft, Launder and Suga (1996) and it is implemented in the finite volume commercial code ANSYS-FLUENT v. 6.3.26 trough specific additional subroutines.

MODELLING EQUATIONS

In the model developed by Jones and Launder in 1972 , the k-ε model, the closure problem is solved trough two differential transport equations: one for k, the turbulent kinetic energy, and a second one for ε, the turbulent dissipation rate.

Non-Linear Eddy-Viscosity Model

The Boussinesq hypothesis limitations have motivated the researchers' effort to find a more accurate form for defining the relation between the Reynolds-stress tensor and the mean flow quantities (mainly the strain rate tensor).
In the Non Linear Eddy Viscosity Model (NLEVM) formulation explicit definitions of Reynolds stress components are given.
A second order formulation for the stress-strain relationship is enough to capture the characteristic of complex flow. The following equation shows a quadratic formulation for the stress-strain relationship common to all EARSM:

$$-\rho\overline{u_i u_j}=\frac{2}{3}\rho k\delta_{ij}-\mu_t S_{ij}+C_1\mu_t\frac{k}{\varepsilon}\left[S_{ik}S_{kj}-\frac{1}{3}\delta_{ij}S_{kl}S_{kl}\right]+C_2\mu_t\frac{k}{\varepsilon}\left[\Omega_{ik}S_{kj}+\Omega_{jk}S_{ki}\right]+$$
$$+C_3\mu_t\frac{k}{\varepsilon}\left[\Omega_{ik}\Omega_{kj}-\frac{1}{3}\delta_{ij}\Omega_{kl}\Omega_{kl}\right]$$

where:

$$S_{ij}=\left(\frac{\partial U_i}{\partial x_j}+\frac{\partial U_j}{\partial x_i}\right)\quad \Omega_{ij}=\left(\frac{\partial U_i}{\partial x_j}-\frac{\partial U_j}{\partial x_i}\right)$$

In this work, the implementation by Shih-Zhu-Lumley model is used. This second order model is characterized by an explicit relationship linking to the strain rate and the vorticity tensors:

$$C_\mu=\frac{2/3}{A_1+S+\alpha\Omega}$$

where:

$$S=\frac{k}{\varepsilon}\sqrt{\frac{1}{2}S_{ij}S_{ij}}\quad \Omega=\frac{k}{\varepsilon}\sqrt{\frac{1}{2}\Omega_{ij}\Omega_{ij}}$$

and A_1 and α value are picked from different set of available data.

In the current study, the structure of Shih-Zhu and Lumley has been used coupled with constants (A_1=3.9, α=0) taken from an experimental campaign done at the THTLab of the University of Tokyo.

Model	C_1	C_2	C_3
Shih, Zhu and Lumley	$\frac{0.75/C_\mu}{1000+S^3}$	$\frac{3.8/C_\mu}{1000+S^3}$	$\frac{4.8/C_\mu}{1000+S^3}$

It is interesting to note that all the three coefficients multiplying the second order factors in the quadratic formulation (C_1, C_2, C_3), as reported in previous table, present the same structure: they are proportional to the inverse of C_μ and to the inverse of a cubic function of S. This structure, together with a non constant definition of C_μ, derives from respecting realizability in the second order formulation of the stress-strain relationship.

Experimental references

The validation with the Square Duct is referred to experimental data of Cheesewright et al.. The Reynolds number based on the mean velocity flow U_b (centreline velocity) is:

$$Re_h=\frac{2\cdot h\cdot U_b}{\upsilon}=4410$$

where h is the half height of the duct. The domain is characterized by periodic boundary conditions.

For the Backward Facing Step experimental data are taken from Jovic and Driver already used by Moin in order to validate their DNS simulation. Jovic and Driver have measured the profiles of velocity in a double expansion duct, with an expansion ratio equal to 1.2. The step is 0.96 cm high and is positioned at 40 h from the entrance of the duct and followed from a section of length 140 h. The Reynolds number, based on the height of the step and the free stream velocity U_0=7.72 m/s measured 3 cm upstream the step, is:

$$Re_h=\frac{h\cdot U_0}{\upsilon}=5100$$

The profiles of velocity are carried out with a Laser Doppler Velocimeter (LDV).

It is relevant to note that the Low Reynolds numbers used in this experimental setting contribute to render them very severe test cases for turbulence modeling since transition is involved.

RESULTS ANALISIS

The flow in the Square Duct is characterized by the presence of the secondary motion developing in a plane, orthogonal to the streamwise velocity.

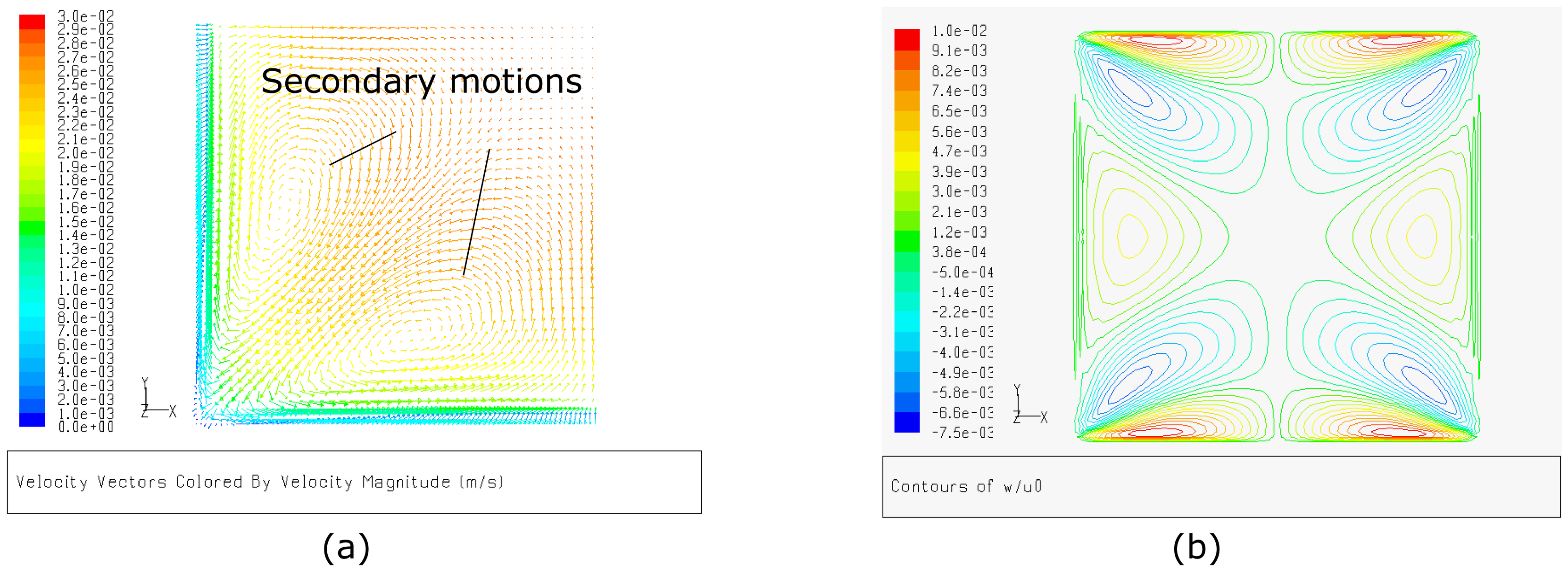

Fig. 1 Velocity vectors (a; only a quarter) and secondary motions (b) field in z=4h section

In Figure 1a, the velocity vectors in a quarter of a section orthogonal to the streamline velocity are shown. The symmetric secondary flows can be easily individuated.
Figure 1b shows that the flow, in a plane orthogonal to the direction of the motion, is characterized by the appearance of secondary flows with a magnitude not greater than 1% of the streamwise velocity (U_b).
It comes out that, as it is well know, the models based on the linear Boussinesq hypothesis can not correctly simulate such secondary flow, while a quadratic formulation proves to be adequate for capturing this feature.

In the Backward Facing Step, the step causes the separation of the boundary layer and its reattachment downstream. The separation point is fixed in correspondence of the abrupt expansion at the top of the step.

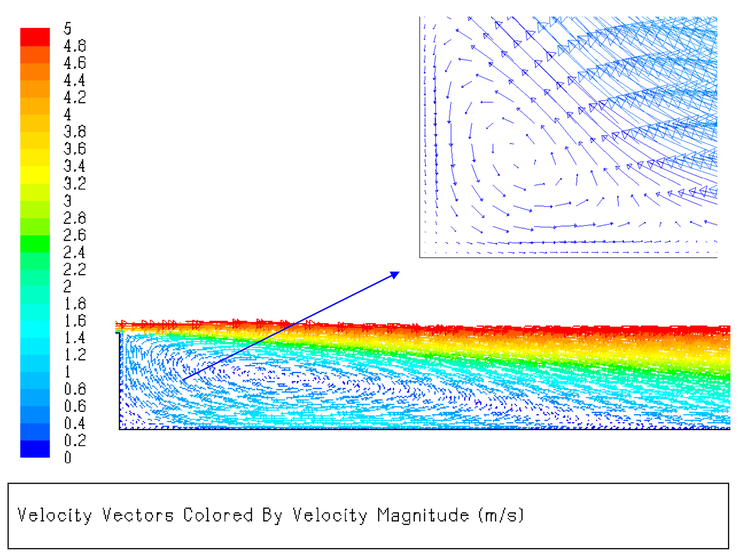

Fig. 2 Velocity vectors

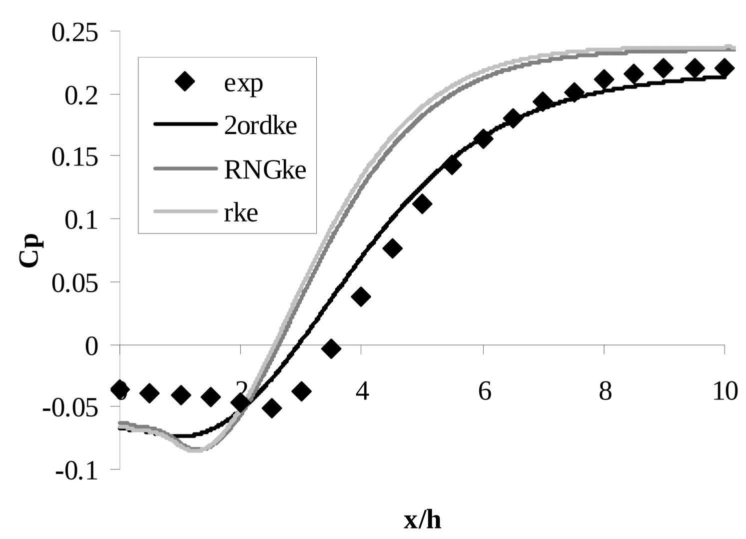

Fig. 3 Coefficient C_p

In figure 2, the velocity vectors show the presence of the counter rotating vortices. In the same figure a detail of the smaller secondary vortex in proximity of the wall is also shown.
The proposed model, 2ordke, is compared with the linear native model of the code (realizable k-ε, rke, and RNG k-ε, RNGke) and with experimental data, exp. The length of reattachment, measured as the distance from the step where the wall shear stresses go to zero, has been used for validating the model. All the models underestimate the experimental value, but the error is within the 10% .

Models	**Reattachment lengths**
Rke	5.6 h
RNGke	5.8 h
2ordke	5.6 h
Experimental data	
Jovic e Driver	6.0 h

In figure 3 the comparison between the numerical data and the experimental data for the Static Pressure Coefficient is given.
Such coefficient is defined as:

$$C_p=\frac{2(p-p_0)}{\rho U_0^2}$$

where p it is the value of the static pressure at the wall while p_0 it is a static reference pressure located at x/h=-5.1 upstream the step.
The coefficient is plotted up to 10 h downstream the step where the recompression is nearly completed.
It is interesting to note how the second order model shows a better agreement with the experimental data in the zone of recirculation and in the area of recompression.

CONCLUSION

The implemented model with a quadratic proposal for the stress-strain relationship is able to predict the secondary flow in a Square Duct and also to improve the prediction of the velocity profile in the zone of recirculation downstream the step.
The proposed model has an intrinsic limitations when used at low Reynolds, associated to the wall treatment approached natively implemented in the commercial code and used in this work. This limitation is relevant in all the wall bounded flow and particularly effecting flow field presenting regions at low Reynolds. A low Reynolds formulation, with the introduction of proper dumping function, would add an additional benefit to the quadratic formulation of the stress-strain relationship and is currently under implementation by the authors.